

# Project MATHEMATICS!

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## Project description

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*Project MATHEMATICS!* produces [videotape-and-workbook modules](#) that explore basic topics in high school mathematics in ways that cannot be done at the chalkboard or in a textbook. The tapes use live action, music, special effects, and imaginative computer animation. [They are distributed on a nonprofit basis.](#)

The goal of the project is to attract young people to mathematics through high-quality instructional modules that show mathematics to be understandable, exciting, and eminently worthwhile. Each module consists of a videotape together with a workbook, and explores a basic topic in mathematics that can be easily integrated into any existing high school or community college curriculum. The modules are crafted to encourage interaction between students and teachers.

More than 10 million students have seen one or more of the videotapes. They have been [enthusiastically received](#) by teachers and students nationwide and have captured first-place [honors](#) at many major film and video festivals.

All tapes have closed captions for the hearing impaired. They are also available in PAL format for use abroad.

Project modules were produced by [Tom M. Apostol](#) and James F. Blinn at the [California Institute of](#)

# The modules

The following modules are currently available

- [The Theorem of Pythagoras](#) Several engaging animated proofs of the Pythagorean theorem are presented, with applications to real-life problems and to Pythagorean triples. The theorem is extended to 3-space, but does not hold for spherical triangles.
- [The Story of Pi](#) Although pi is the ratio of circumference to diameter of a circle, it appears in many formulas that have nothing to do with circles. Animated sequences dissect a circular disk of radius  $r$  and transform it to a rectangle of base  $\pi \cdot r$  and altitude  $r$ . Animation shows how Archimedes estimated pi using perimeters of approximating polygons.
- [Similarity](#) Scaling multiplies lengths by the same factor and produces a similar figure. It preserves angles and ratios of lengths of corresponding line segments. Animation shows what happens to perimeters, areas, and volumes under scaling, with various applications from real life.
- [Polynomials](#) Animations show how the Cartesian equation changes if the graph of a polynomial is translated or subjected to a vertical change of scale. Zeros, local extrema, and points of inflection are discussed. Real-life examples include parabolic trajectories and the use of cubic splines in designing sailboats and computer-generated teapots.
- [Sines and Cosines, Part 1 \(Periodic functions\)](#) Sines and cosines occur as rectangular coordinates of a point moving on a unit circle, as graphs related to vibrating motion, and as ratios of sides of right triangles. They are related by reflection or translation of their graphs. Animations demonstrate the Gibbs phenomenon of Fourier series.
- [Sines and Cosines, Part 2 \(Trigonometry\)](#) This program focuses on trigonometry, with special emphasis on the law of cosines and the law of sines, together with applications to The Great Survey of India by triangulation. The history of surveying instruments is outlined, from Hero's dioptra to modern orbiting satellites.
- [Sines and Cosines, Part 3 \(Addition formulas\)](#) Animation relates the sine and cosine of an angle with chord lengths of a circle, as explained in Ptolemy's *Almagest*. This leads to elegant derivations of addition formulas, with applications to simple harmonic motion.
- [The Tunnel of Samos](#) This video describes a remarkable engineering work of ancient times: excavating a one-kilometer tunnel straight through the heart of a mountain, using separate crews that dug from the two ends and met in the middle. How did they determine the direction for excavation? The program gives Hero's explanation (ca. 60 A.D.), using similar triangles, as well as alternate methods proposed in modern times.
- [Teachers Workshop](#) This 28-minute tape, accompanied by a 90-page transcript, contains excerpts from a two-day workshop held in 1991 for teachers who have successfully used project materials in their classrooms.

- [Project MATHEMATICS! Contest](#) In 1994 *Project MATHEMATICS!* conducted a contest open to all teachers who had used project materials in their classrooms. Entries were judged on the basis of innovative and effective use of the materials. This videotape, accompanied by a 30-page booklet, shows the classroom implementation of the entries of the first-place winners.
- [Early History of Mathematics](#) This 30-minute videotape traces some of the landmarks in the early history of mathematics--from Babylonian clay tablets produced some 5000 years ago, when calendar makers calculated the onset of the seasons--to the development of calculus in the seventeenth century.

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## Distribution network

The modules, most of which were produced under National Science Foundation grants, are issued on a nonprofit basis. Project materials are distributed through an extensive distribution network consisting of

- [34 State Departments of Education](#)
- [The Caltech Bookstore](#)
- [The Mathematical Association of America](#)
- [The NASA Educator Resource Center Network](#)
- [Science Screen Report](#)
- [Foreign Distribution](#)

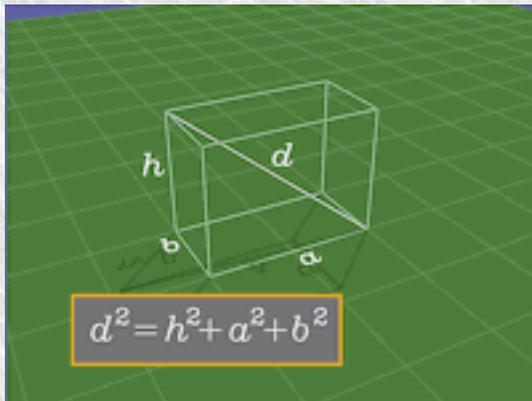
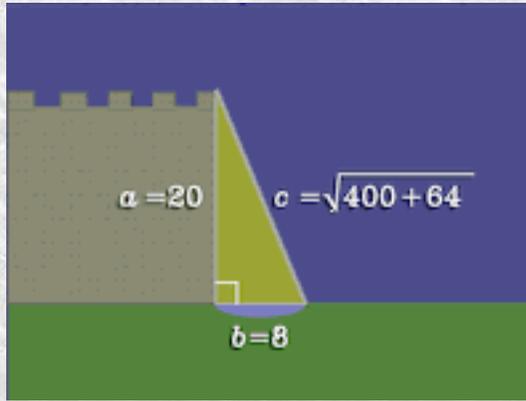
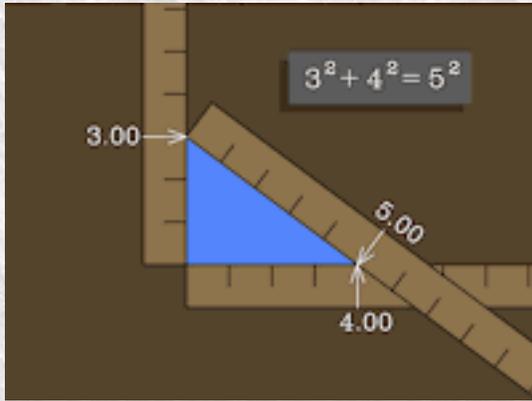
Modules may also be copied freely in the USA for educational purposes under the conditions described on the cassette label. The label reads: "You may reproduce, distribute, perform and display copies of this copyrighted work in the U. S. A. for non-commercial purposes, provided that each copy shall consist of only the entire contents hereof, including this label verbatim, and provided further that no compensation or remuneration, direct or indirect, may be received therefrom."

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For questions concerning the project contact: [Tom Apostol](#)

# The Theorem of Pythagoras



## Video Segments



1. Three questions from real life



2. Discovering the Theorem of Pythagoras



3. Geometric interpretation



4. Pythagoras



5. Applying the Theorem of Pythagoras



6. Pythagorean triples



7. The Chinese proof



8. Euclid's elements



9. Euclid's proof



10. A dissection proof



11. Euclid's Book VI, Proposition 31



12. The Pythagorean Theorem in 3D

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## Contents

The program begins with three real-life situations that lead to the same mathematical problem:

Find the length of one side of a right triangle if the lengths of the other two sides are known.

The problem is solved by a simple computer-animated derivation of the Pythagorean theorem (based on similar triangles):

In any right triangle, the square of the hypotenuse is equal to the sum of the squares of the two legs.

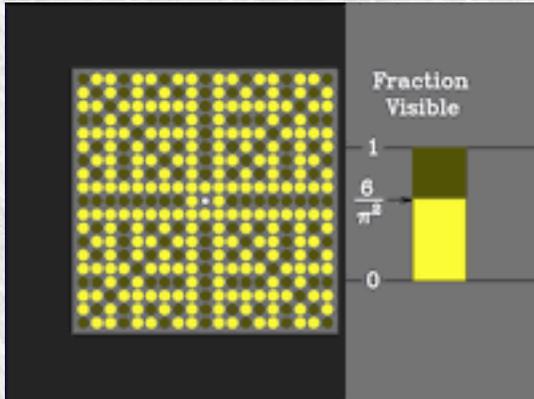
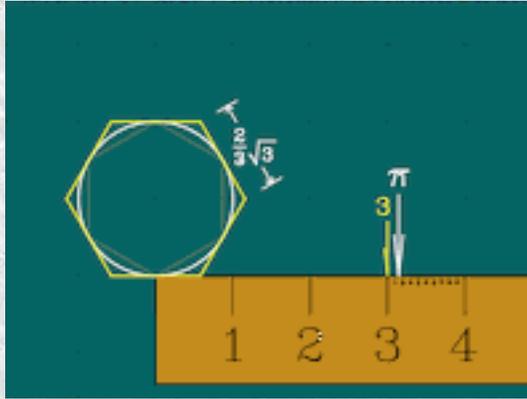
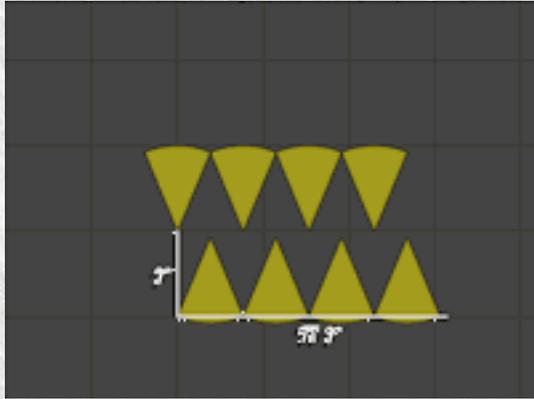
The algebraic formula  $a^2 + b^2 = c^2$  is interpreted geometrically in terms of areas of squares, and is then used to solve the three real-life problems posed earlier. Historical context is provided through stills showing Babylonian clay tablets and various editions of Euclid's *Elements*. Several different computer-animated proofs of the Pythagorean theorem are presented, and the theorem is extended to 3-space.

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# The Story of Pi



## Video Segments



1. What is the number pi?



2. Some uses of pi



3. Early history of pi



4. A discovery of Archimedes



5. Computation of pi



6. Further uses of pi



7. Recap

# Contents

The program opens with a reporter interviewing young people, asking "What can you tell me about the number pi?" Each person gives a different answer, some of which are only partially correct.

The program defines pi as the ratio of circumference to diameter of a circle, and shows how pi appears in a variety of formulas, many of which have nothing to do with circles. After discussing the early history of pi, the program invokes similarity to explain why the ratio of circumference to diameter is the same for all circles, regardless of size. This ratio, a fundamental constant of nature, is denoted by the Greek letter pi, so that  $2\pi r$  represents the circumference of a circle of radius  $r$ .

Two animated sequences show that a circular disk of radius  $r$  can be dissected to form a rectangle of base  $\pi r$  and altitude  $r$ , so the area of the disk is  $\pi r^2$ , a result known to Archimedes. Animation shows the method used by Archimedes to estimate pi by comparing the circumference of a circle with the perimeters of inscribed and circumscribed polygons.

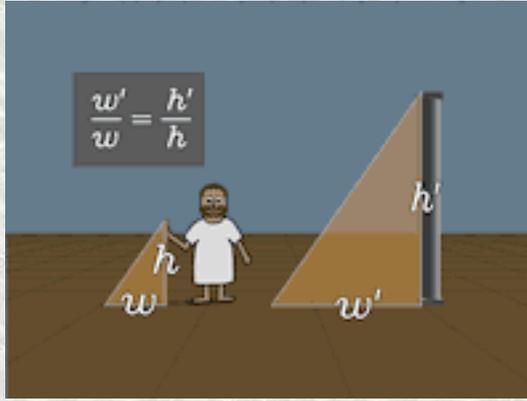
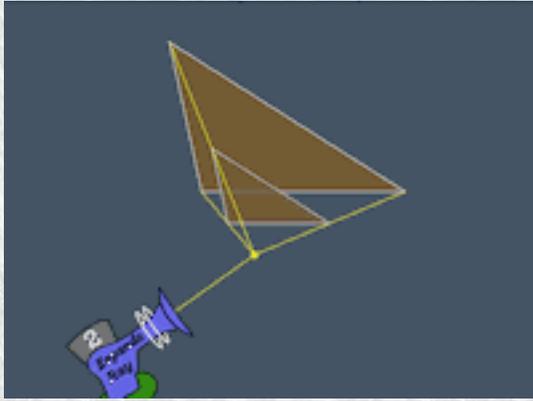
The next segment describes different rational estimates for pi obtained by various cultures, and points out that pi is irrational. After demonstrating the appearance of pi in probability problems, the program returns briefly to the reporter, who interviews the students again, asking, "Now what can you tell me about pi?" This time, each student gives a different correct statement about pi. The concluding segment explains that major achievements in estimating pi represent landmarks of important advances in the history of mathematics.

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# Similarity



## Video Segments



1. Shape and size



2. Similar triangles



3. Applications of similarity



4. Similar polygons and solids



5. Internal ratios of similar figures



6. Perimeters of similar figures



7. Areas of similar figures



8. Volumes of similar figures



## 9. Applications to biology

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# Contents

The opening scene shows various objects from real life having the same shape but not necessarily the same size. The narrator asks ``Can we construct a figure with the same shape as another?" A triangle is moved to various positions by translating it, rotating it, or flipping it over. They are congruent because they have not only the same shape but also the same size.

To change size without changing shape, scaling is introduced. Scaling multiplies lengths of all line segments by the same number and produces a similar figure. Similarity preserves angles and ratios of lengths of corresponding line segments.

Applications show how Thales might have used similarity to find the height of a column and of a pyramid by comparing lengths of shadows. Another application of similarity explains why the sum of the angles in any triangle is a straight angle.

Similarity is discussed for more general polygons and for three-dimensional objects. Animation shows what happens to perimeters, areas, and volumes under scaling, with illustrations from real life.

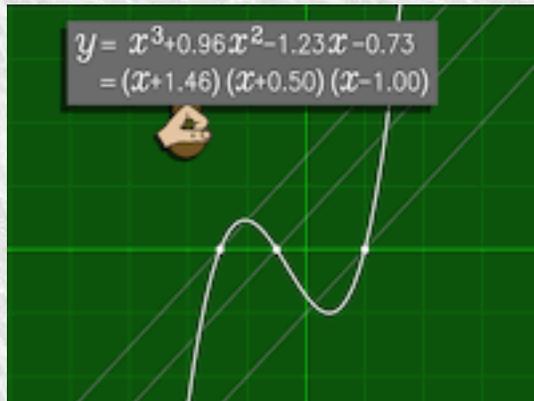
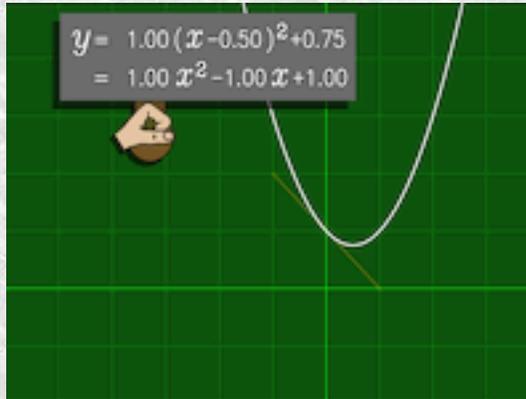
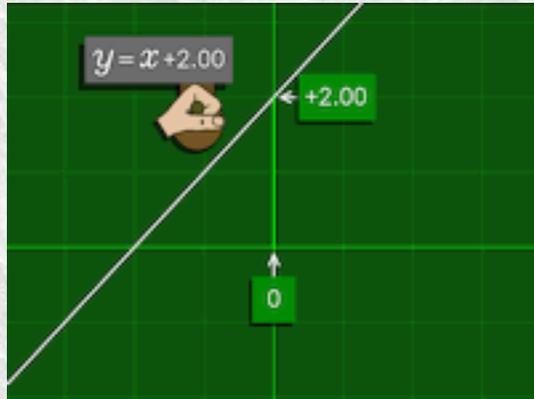
Similarity is the basis of all measurement. It reveals the secret of map making and scale drawings, and also explains some aspects of photographic images. Similarity helps explain why a hummingbird's heart beats so much faster than a human heart, and why it is impossible for a small creature such as a praying mantis to become as large as a horse.

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# Polynomials



## Video Segments



1. Polynomials in real life

2. Linear polynomials



3. Quadratic polynomials

4. Intersections of lines and parabolas

5. Cubic polynomials

6. Polynomials of higher degree

7. Calculation of polynomials

## Contents

The program opens with examples of polynomial curves that appear in real life, including parabolic

trajectories and the use of cubic splines in designing sailboats.

Polynomials are systematically classified by degree. Linear polynomials are discussed first; their graphs are straight lines of various slope.

Quadratic polynomials are discussed next. Their graphs are parabolas, the prototype being the graph of  $y = x^2$ . Animation shows how the Cartesian equation changes when the curve is translated vertically or horizontally or subjected to a vertical change of scale.

Cubic polynomials are treated next, with discussion of zeros, local maxima and minima, and points of inflection. There are three prototypes  $y = x^3$ ,  $y = x^3 + x$ , and  $y = x^3 - x$ . All cubics can be obtained by horizontal or vertical translation or by horizontal or vertical change of scale, or by taking mirror images.

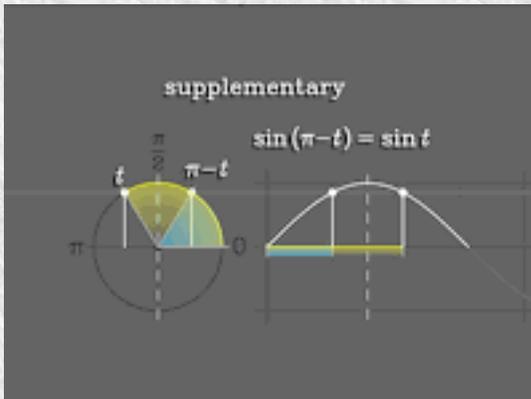
A similar discussion is given for quartics and higher degree polynomials, all of which have infinitely many prototypes.

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# Sines and Cosines, Part I (Periodic Functions)



## Video Segments

1. Circular motion and sine waves
2. Symmetry of sine waves
3. Sine waves and sound
-  4. Periodic waves
5. Sines and cosines as ratios
6. Preview of Sines and Cosines, part II

## Contents

Sines and Cosines, Part I shows how sines and cosines arise in different contexts: As the rectangular coordinates of a point moving on a unit circle, as graphs related to vibrating motion (illustrated by musical instruments), and as ratios of sides of right triangles.

Reflecting the sine curve about various lines reveals simple properties of the sine function, for example,  $\sin(-t) = -\sin t$ ,  $\sin(\pi - t) = \sin t$ ,  $\sin(\pi + t) = -\sin t$ . Reflection of the sine curve about the line  $t = \pi/4$  generates a new curve, called a cosine curve, given by  $\cos t = \sin(\pi/2 - t)$ .

Periodic waves are discussed, and the tape illustrates Fourier's remarkable discovery that all periodic functions are linear combinations of sines and cosines. Historical background of trigonometry is

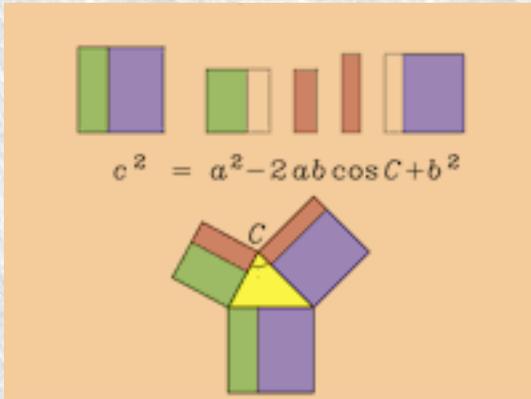
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# Sines and Cosines, Part II (Trigonometry)



## Video Segments

1. Trigonometry
2. Sines, cosines and the Pythagorean Theorem
3. The law of cosines
4. Applying the law of cosines
5. The law of sines
6. Applying the law of sines
7. Surveying by triangulation
8. Preview of parts III and IV



## Contents

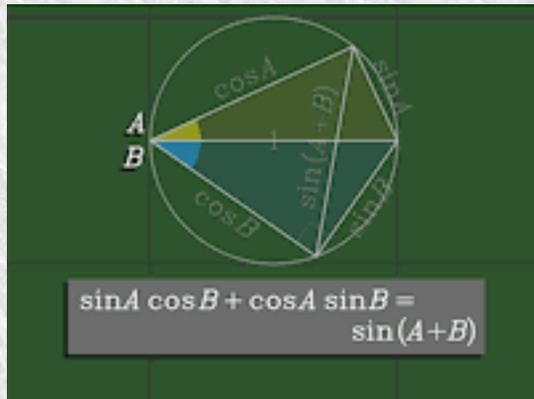
Sines and Cosines, Part II (Trigonometry) focuses on the use of sines and cosines in trigonometry, with special emphasis on the law of cosines and the law of sines. They enable us to find all parts of a triangle if three parts are known, and at least one of them is a side. Applications are described in astronomy, navigation, and surveying by triangulation.

One of the major triumphs of surveying by triangulation is the Survey of India, which took more than a century to complete. The program describes how the survey was done and how it determined the height of Mt. Everest. The program also outlines a brief history of surveying instruments, from the dioptra of ancient times to orbiting satellites of modern times.

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# Sines and Cosines, Part III (Addition Formulas)



## Video Segments

1. Sines and chord lengths
2. Addition formula for sines
3. Ptolemy's theorem on quadrilaterals
4. Applications of the addition formulas
5. Sines and cosines of special angles
6. Application to simple harmonic motion
7. Preview of part IV
8. Alexandria--Center of Hellenistic Culture



## Contents

Sines and Cosines, Part III (Addition Formulas) relates the sine and cosine of an angle with lengths of chords of a circle, as expounded in Claudius Ptolemy's *Almagest*. This leads to simple derivations of the addition formulas for determining the sine and cosine of a sum of two angles. One application shows that a combination of a sine wave with a cosine wave of the same frequency is another sine wave, possibly shifted. This property plays an important role in the study of simple harmonic motion.

The program also outlines a brief history of the city of Alexandria, founded by Alexander the Great in 331 B.C. His successors created a center of Hellenistic culture in Alexandria that attracted many of the

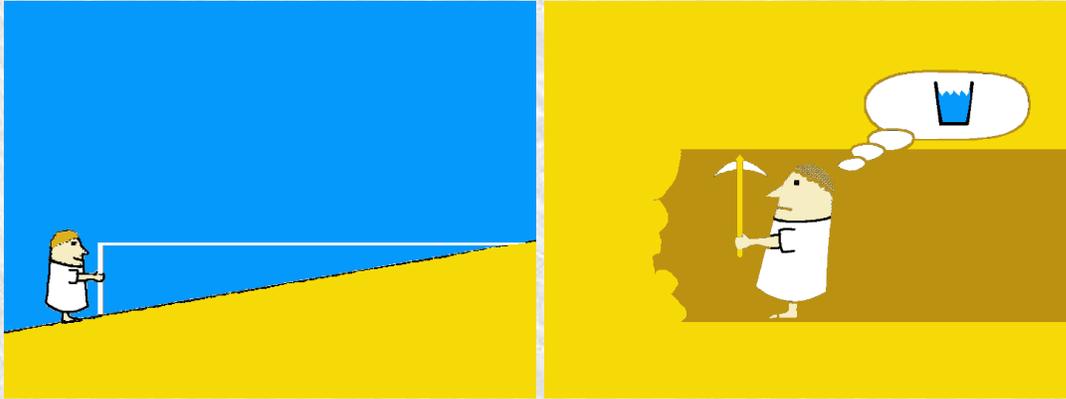
greatest mathematical scholars of antiquity, including Euclid, Apollonius, Archimedes, Eratosthenes, Hero, Pappus, and Claudius Ptolemy.

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# The Tunnel of Samos



## Video Segments

1. The quest for water
2. Bringing water to Samos
3. Excavating tunnels from both ends
4. Hero's explanation
5. Another explanation
6. Completing the tunnel
7. Later history of the tunnel



## Contents

The water supply of the principal city on the island of Samos in ancient Greece was inadequate for its growing population, but there was an ample supply in the mountains. To bring water from the mountains to the city, a one-kilometer tunnel was dug in the 6th century B.C. through a large hill of solid limestone. The tunnelers worked from both ends and met in the middle, more or less as planned. This module shows how similar triangles probably were used to determine the correct direction for tunneling. The workers who carved the tunnel with primitive tools met at the center with an error less than .15% of the length, a remarkable achievement for that era. The module explains why some error could be expected. It also shows that the problem of delivering fresh water to large populations has been an ongoing human endeavor since ancient times.

After centuries of neglect the tunnel became lost until it was rediscovered in 1882 in a relatively good

state of preservation. It contained artifacts dating back to the Roman and Byzantine eras. Shortly thereafter the German archaeologist Ernst Fabricius surveyed the tunnel and published a full description.

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# The Teachers Workshop



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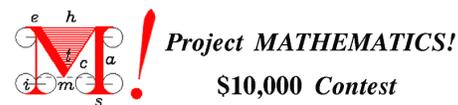
This tape contains excerpts from a two-day workshop held in mid-August 1991 for teachers who had successfully used Project MATHEMATICS! materials in their classrooms, specifically [The Theorem of Pythagoras](#), [The Story of Pi](#), [Similarity](#), and [Polynomials](#). Broader issues in mathematics education were also discussed: for example, the role of visualization, the use of manipulatives, and different learning and teaching styles. The teacher participants explain how Project materials have been used in a variety of classroom settings, from grades 8 to 13, and how they have adapted to new ideas and new technology in creative ways that enhance their teaching and motivate their students to become excited about learning mathematics.

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# Project MATHEMATICS! Contest



sponsored jointly by

**The Hewlett-Packard Company  
& The Intel Foundation**

Awards of \$1,000 will be made to each of five teachers who, in the opinion of a judging committee, demonstrate outstanding success through innovative and effective use of *Project MATHEMATICS!* videotapes and workbooks in the classroom.

Each grantee's school will receive an additional award of \$1,000 to be used in a manner determined by the winning teacher.

*Project MATHEMATICS!* will produce a 30-min videotape showing implementation of each prize-winning proposal, together with a booklet describing proposals of other entrants. Each entrant will receive a complimentary copy of this videotape and the accompanying booklet.

*Project MATHEMATICS!* conducted a contest in 1994 open to all teachers who had used project materials (videotapes and workbooks). Entries were judged on the basis of innovative and effective use of these materials in the classroom. Through the generosity of The Hewlett-Packard Company and The Intel Foundation eight prizes totaling \$13,000 were awarded. They consisted of five first-place awards of \$1,000 each to teachers, with an additional \$1,000 presented to the awardee's school to be used in a manner determined by the awardee. In addition, second-place awards of \$500 each went to three teachers, with another \$500 presented to the awardee's school to be used in a manner determined by the awardee. The names of the winners are listed [on the next page](#).

A videotape was prepared showing classroom implementation of the entries of the first-place winners. This booklet describes these entries in more detail and also describes the entries of the second-place winners. The Hewlett-Packard Company and The Intel Foundation jointly provided financial support to produce the videotape and the booklet and to distribute 1,000 complimentary copies to teachers nationwide.

Entries were received from various parts of the country, from Canada, and from overseas. Grade levels ranged from grade 8 in middle school to first-year community college. The contest entries show that teachers who are free to experiment can adapt to new ideas and new technology in creative ways that enhance their teaching and motivate their students to become excited about learning mathematics.

All participants are grateful to The Hewlett-Packard Company and to The Intel Foundation for generous support that made it possible to share in this truly educational experience and to extend its benefits to the mathematical community at large.

## Contest Awardees

Five first-place awards of \$1,000 each (listed in alphabetical order). An additional \$1,000 was presented

to the awardee's school to be used in a manner determined by the awardee.

Suzanne Jacobsen,

8th grade teacher at Jericho Middle School in Jericho, New York. For a series of activities that use a variety of learning styles to facilitate student understanding of the number pi.

Tom Janssens and Sandy Lofstock,

teachers at California Lutheran University in Thousand Oaks, California. For hands-on activities related to sines and cosines.

Edna R. Mangaldan and Alicia D. Pambid,

10th grade teachers at Manuel A. Roxas High School in Paco, Manila, Philippines. For student activities, recorded on videotape, related to the modules on Polynomials, the Theorem of Pythagoras, The Story of Pi, Similarity, and Sines & Cosines, Part I.

Sue Stetzer,

11th grade teacher at J. R. Masterman School in Philadelphia, Pennsylvania. For a student newspaper project including several features having to do with trigonometry.

Ron Woggon,

teacher in grades 9 through 12 at John A. Rowland High School in Rowland Heights, California. For a series of student produced animated videotapes motivated and inspired by Project MATHEMATICS!.

Three second-place awards of \$500 each (listed in alphabetical order). An additional \$500 was presented to the awardee's school to be used in a manner determined by the awardee.

Robin-Lynn Clemmons,

8th grade teacher at Holy Innocents' Episcopal School in Atlanta, Georgia. For hands-on projects related to the Theorem of Pythagoras.

Steve Lifer,

teacher in grades 9 through 11 at Lexington High School in Lexington, Ohio. For student activities simulating questions from real life as presented in the module on The Theorem of Pythagoras.

Joanne Yau,

geometry teacher at Galena High School in Reno, Nevada. For student activities related to the modules on Similarity and The Theorem of Pythagoras.

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# Early History of Mathematics



## Video Segments



1. Introduction
2. From Euclid to the Seventeenth Century
3. From Scratch Marks to Number Systems
4. From Numerology to Number Theory
5. The Pythagorean Theorem
6. A Shocking Discovery
7. Pi Through the Ages
8. From Astronomy to Trigonometry
9. From Archimedes to Fermat and Descartes
10. The Race for the Calculus

## Contents

Two introductory segments give an overview of the program. The first outlines some of the important developments in the period from 3000 B.C. to 300 B.C., which culminated with the publication of Euclid's *Elements*. The second gives an outline of some of the landmark achievements from 300 B.C. up to the events that led to the development of calculus. The remaining segments describe in more detail some of the important highlights on the road to calculus. They discuss number systems developed in

different cultures; numerology or number mysticism as practiced by the Pythagoreans; the origin of number theory as an outgrowth of numerology; the Pythagorean Theorem for right triangles and how it led to the discovery of irrational numbers; the golden age of Hellenistic mathematics as it flourished in the ancient city of Alexandria; the multi-cultural search for an understanding of the number pi and the landmark contributions to this effort by Archimedes; how astronomy gave birth to trigonometry; and other developments such as algebra and analytic geometry that eventually led to the calculus.

The program also includes some mathematical derivations in animated form: proofs of the Pythagorean Theorem, a new geometric proof of the irrationality of the square root of two, two methods for calculating the area of a circular disk, and the Archimedes method for estimating pi.

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## Some Unsolicited Comments from Viewers:

"For most of the thirty years since I was graduated from high school, I've held the concepts of Sines and Cosines at arms length. I knew that they were good for something, just as the odd wrench that you find at the bottom of the toolbox must fit something, but as to what, I was clueless.

An acquaintance suggested that I view the Project MATHEMATICS video collection on the subject. As a result, my teeth are whiter, my cat is friendlier, and my mileage has improved drastically. Okay, maybe not -- but I now know more about \*why\* people care about Sines, and how they might use them. I'm sure my knowledge is a pale reflection of reality, but I am pleased to have it, nevertheless. I owe it to the Project Mathematics videos, and I thank you for them."

"...I just wanted to send you a quick note of thanks for the elegant and captivating way you've managed to portray mathematics in your "Project MATHEMATICS!" videos.

I was an innocent bystander, just checking in to a Seattle hotel for a routine business trip, when I flipped on the TV to lull me as I've done hundreds of times before. This time though, as I idly scanned the channels, I landed on a station showing one of your videos, and the simple, engaging way it managed to explain trig was eye-opening. I'd forgotten much if not all of my high school and college trig, but I found myself sitting open-mouthed on the edge of my chair and devouring the whole thing. I couldn't take my eyes off the screen ...your video reminded

me why I like learning for learning's sake.

Don't suppose you know of any similar series for physics, chemistry and/or biology do you? I'd rather watch your stuff and others of its ilk than a copy of "Armageddon" any day."

"...I'm an adult who has avoided mathematics most of my life because I never had teachers who explained what in the world was going on. I assumed I was just an idiot, but since I've worked at research facilities as a technical secretary and have seen the joy that mathematicians, engineers and physicists seem to get out of their equations, I wondered what they knew that I didn't know. Your program changed my perception!

I must admit that the classical music that's played on the program lured me into watching your video, and through sheer curiosity I couldn't believe that I was understanding, for the first time in my life, what the world of mathematics was all about. I just love the visual effects that explain what the equation is representing. My husband and I totally agree that we would have developed a definite interest for math a lot sooner if it had been explained to us in this manner.

I had to write to you and tell you how much your program has influenced my life and has actually sparked a wonderful interest in learning algebra, geometry, trig and calculus. The visual effects have opened a whole new world to me and I can finally pursue areas that require mathematical skill because I'm no longer afraid of "scary ol' math."

Thank you for changing my world!"

"...I am currently (as I have been for the last 21 years) teaching mathematics in Texas, just a hop, skip and jump from NASA. I have seen bits and pieces of the videos on our local NASA channel, but had not seen the lead in until one of our teachers, attending a workshop, secured a couple of them. I am so impressed. I really enjoyed them myself, and feel sure a number of my students and those of the other 14 math teachers could profit from the use of the videos in class. The voice over stated that there were workbooks available and if they are the same quality as the videos --well, what can I say?

Please send me the information regarding the videos that are available and any and all supporting materials, too. I am looking forward to hearing from you soon as "Sines and Cosines, Part III *is* my pre-calculus chapter 6 -- but waaay better."

"I am a computer engineering student at the University of Minnesota. I got home from the lab tonight and flipped on the TV before going to bed. I saw your math program dealing with trigonometry and the various trig identities for sin and cos. I would like to say two things. Your math videos have proved to me that done correctly, television can actually be a useful teaching method and that in the brief span that I watched I was actually able to find out why something works rather than just how.

This has always been important to me. Although as an engineer I am supposed to be this all-knowing math machine, there are many times when I finish a course and still don't really know why things work the way they do. Maybe because I am supposed to be a math genius this bothers me more than most people. I was educated in a very strict liberal arts environment and never had very much math in high school. I took trigonometry in college and am still taking math now. I'll be going into multi-variable calculus. Although I did well, I was always left feeling unfulfilled by not really knowing why these identities (which we had to memorize) were what they were. Your videos did an excellent job of illustrating why. It was wonderful to feel the great "A-HA!" experience again. I especially liked how the video progressed from the

addition and subtraction formulas for sin and cos and was able to show how the double angle formulas and the fundamental identity  $\sin^2 + \cos^2 = 1$  can be derived from those. I thought that was pretty slick.

I was so impressed that I would like to purchase these videos. Please let me know how I can get copies. Thank you for your hard work and keep up the good job you guys are doing."

"...I stumbled upon a wonderful presentation on television today which turned out to be from your Project MATHEMATICS! As a high school math teacher, I was impressed by the animation used to present the concepts. I saw two topics today, one on 4th degree equations showing how changes in the equation resulted in changes in the graph, and another on the Pythagorean Theorem. The animation quickly and easily shows relationships in a way I cannot.

Based on the little bit I saw today, I can see a real use for them in the classroom."

"...I recently saw something on the Discovery Channel that was produced by Project Mathematics on Trigonometry. It was produced in 1992, and showed the relationship between trigonometry functions and angles of a triangle.

I was wondering if I could get more information about this series. I am a high school math teacher and will be teaching Algebra I and Algebra II/Trig. this year, and I thought that video would be wonderful to incorporate into my class."

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# Awards won by *Project MATHEMATICS!*

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## *Theorem of Pythagoras*

Gold Medal, 1988 International Film and TV Festival of New York

Gold Apple, 1989 National Educational Film and Video Festival, Oakland

Blue Ribbon, Best of Category, 1989 American Film and Video Festival, Chicago

Electra Certificate, Best of Category, 1989 Birmingham International Educational Film Festival

Gold Cindy, 1989 Cindy Competition, Association of Visual Communicators, Los Angeles

## *The Story of Pi*

Gold Apple, 1990 National Educational Film and Video Festival, Oakland

Red Ribbon, 1990 American Film and Video Festival, Chicago

## *Similarity*

Silver Apple, 1991 National Educational Film and Video Festival, Oakland

## *Sines and Cosines, Part I*

Silver Medal, 1992 New York Festival

## *Sines and Cosines, Part II*

Gold Medal, 1993 New York Festival

Gold Apple, 1994 National Educational Film and Video Festival, Oakland

## *The Tunnel of Samos*

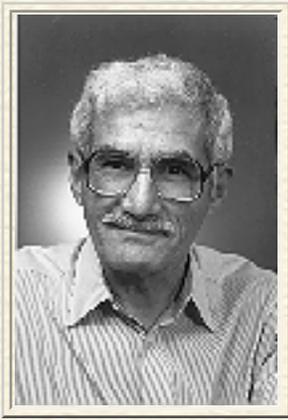
World Medal, 1995 New York Festival

In addition, *Project MATHEMATICS!* was nominated by the Hewlett-Packard Company in 1991 for a **Computerworld Smithsonian Award**, in recognition of its innovative use of information technology.

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# TOM M. APOSTOL

## Professor of Mathematics, Emeritus

### Creator and Project Director, [Project MATHEMATICS!](#)

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B.S. in Chemical Engineering, University of Washington, Seattle, 1944

M.S. in Mathematics, University of Washington, Seattle, 1946

Ph.D. in Mathematics, University of California, Berkeley, 1948

### Research Interests

Analytic number theory, mathematics education

### Recent Publications

- *Introduction to Analytic Number Theory*, corrected fifth printing, Undergraduate Texts in Mathematics, New York-Heidelberg-Berlin: Springer-Verlag, 1998 (338 pages)
- *Modular Functions and Dirichlet Series in Number Theory*, second ed., Graduate Texts in Mathematics **41**, New York-Heidelberg-Berlin: Springer-Verlag, 1990 (204 pages)
- *Linear Algebra, a First Course, With Applications to Differential Equations*, New York: John Wiley & Sons, Inc., 1997 (347 pages)
- *Early History of Mathematics*, Videotape and workbook/study guide, *Project Mathematics!*, Caltech, 2000
- *Irrationality of the Square Root of Two — A Geometric Proof*, Amer. Math. Monthly, 107, Nov. 2000, pp. 834–835
- *Contributions by Charles-Jean de la Vallée Poussin to the Theory of Numbers*, Poussin's Collected Works, v. 1, 2000, Rendiconti del Circolo Matematico di Palermo, pp. 103–116

- *A Visual Approach to Calculus Problems*, Engineering & Science, Vol. LXIII, No. 3, 2000, Caltech, pp. 22–31
  - *Sums of Squares of Distances*, Math. Horizons, November 2001, pp. 21–22 (with Mamikon A. Mnatsakanian)
  - *Subtangents — An Aid to Visual Calculus*, American Mathematical Monthly, June/July 2002, pp. 517–525 (with Mamikon A. Mnatsakanian)
  - *Generalized Cyclogons*, Math Horizons, September 2002, pp. 25–29 (with Mamikon A. Mnatsakanian)
  - *Surprisingly Accurate Rational Approximations*, Mathematics Magazine, 74, October 2002, pp. 307–310 (with Mamikon A. Mnatsakanian)
  - *Tangents and Subtangents Used to Calculate Area*, American Mathematical Monthly, December 2002, pp. 900–909 (with Mamikon A. Mnatsakanian)
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Please contact NASA's Central Operation of Resources for Educators at the following address for information on how to obtain the *Project MATHEMATICS!* materials or to contact the NASA Educator Resource Center located near you:

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Oberlin, OH 44074

440-775-1400

FAX 440-775-1460

E-mail: [nasaco@leeca.org](mailto:nasaco@leeca.org)

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