

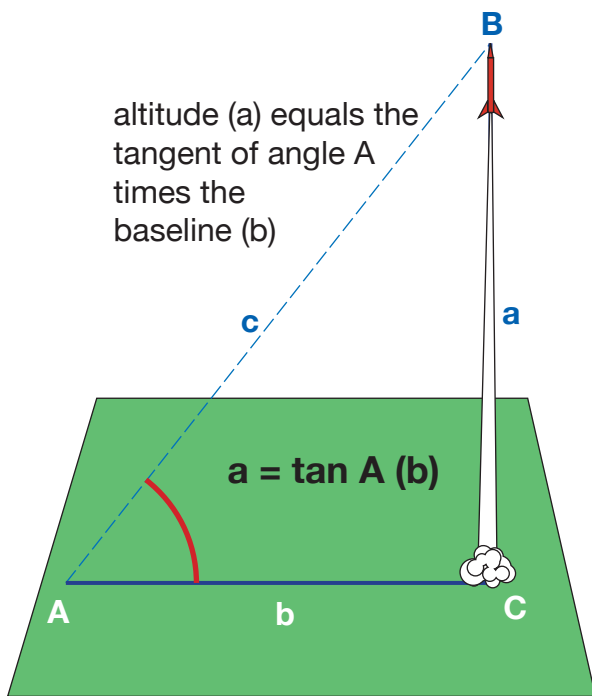
Above and Beyond

Additional Explorations

How High?

Using Mathematics to Estimate Rocket Altitude

Students are excited to learn what altitude their rockets achieve. Altitude tracking is both simple and tricky. If the rocket goes straight up, it is pretty easy to get a good estimate of the altitude. The altitude tracker activity (page 80) provides a simple instrument and instructions for estimating rocket altitudes. A baseline is stretched out from the rocket launch site. The angle to the rocket, just before it starts its fall back to Earth, is measured. The tangent of the



Sample Measurement:

Angle A = 40 degrees

Tangent A = .8391

Baseline b = 25 m

a (altitude) = $\tan A \times 25$ m

$a = 20.97$ m

angle is determined from the tangent table in the tracker activity. The tangent, multiplied by the length of the baseline, gives the altitude.

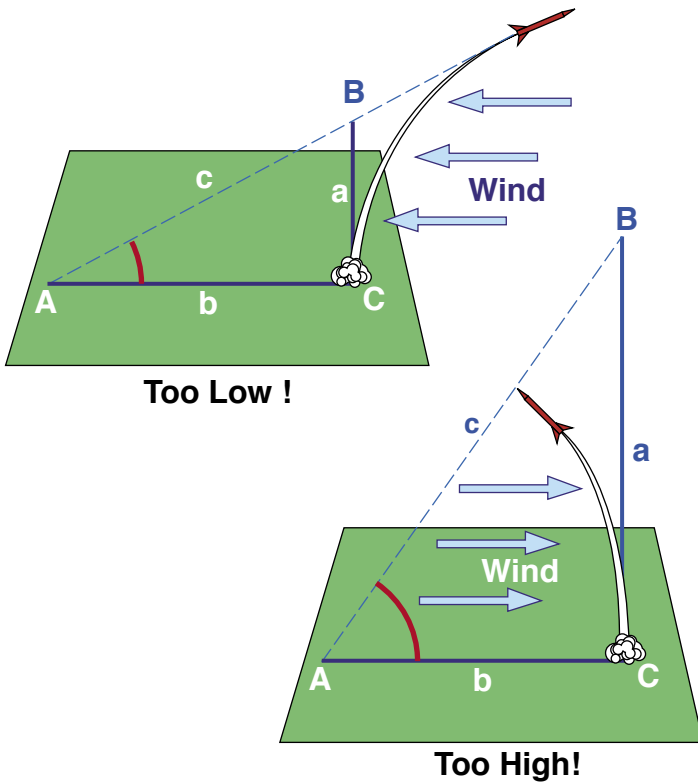
Single station tracking is easy to do. If you have two or more students measure the angle, averaging their estimates can increase accuracy.

Tracking becomes more challenging when rockets stray from straight up. Wind will cause the rocket to drift. Wind pushes the fins away while the nose cone points towards the wind. This causes the rocket to nose into the wind, resulting in larger altitude error estimates.

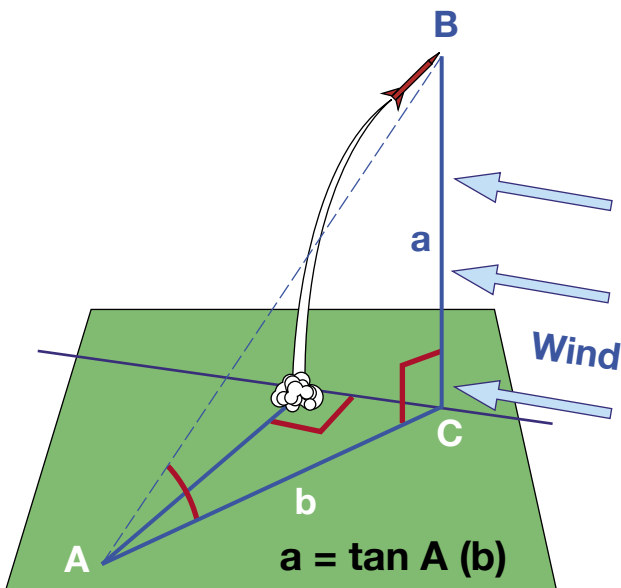
Angle	Tangent	Angle	Tangent
0	.0000	46	1.036
1	.0175	47	1.072
2	.0349	48	1.111
3	.0524	49	1.150
4	.0699	50	1.192
5	.0875	51	1.235
6	.1051	52	1.280
7	.1228	53	1.327
8	.1405	54	1.376
9	.1584	55	1.428
10	.1763	56	1.483
11	.1944	57	1.540
12	.2126	58	1.600
13	.2309	59	1.664
14	.2493	60	1.732
15	.2679	61	1.804
16	.2867	62	1.881
17	.3057	63	1.963
18	.3249	64	2.050
19	.3443	65	2.145
20	.3640	66	2.246
21	.3839	67	2.356
22	.4040	68	2.475
23	.4245	69	2.605
24	.4452	70	2.747
25	.4663	71	2.904
26	.4877	72	3.078
27	.5095	73	3.271
28	.5317	74	3.487
29	.5543	75	3.732
30	.5774	76	4.011
31	.6009	77	4.331
32	.6249	78	4.705
33	.6494	79	5.145
34	.6745	80	5.671
35	.7002	81	6.314
36	.7265	82	7.115
37	.7536	83	8.144
38	.7813	84	9.514
39	.8098	85	11.43
40	.8391	86	14.30
41	.8693	87	19.08
42	.9004	88	28.64
43	.9325	89	57.29
44	.9657	90	---
45	1.000	---	---

Tangent Table from the altitude tracker activity

Single Station



One method for reducing windy day error is to set up the baseline perpendicular to the wind direction. In the diagram that follows, wind causes the rocket to drift to the right. This stretches the baseline a bit but the overall error for the altitude is reduced. Challenge advanced students to come up with a way of determine how much the baseline changes when the rocket drifts to the right.

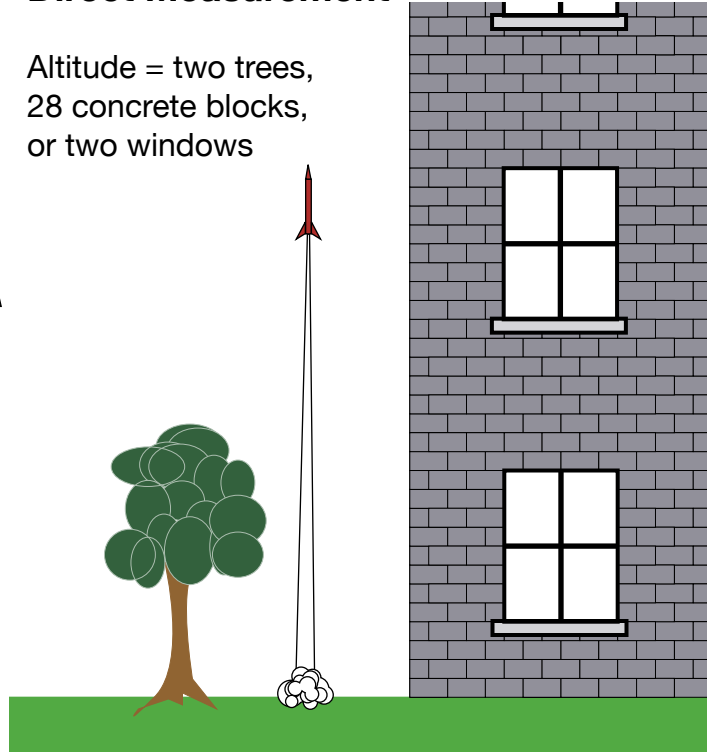


Angle A is reduced but line b is increased by the drift of the rocket.

Wind effects can also be addressed by employing two tracking stations at opposite ends of the baseline. The baseline is stretched up and downwind. Each station measures the altitude the rocket achieves. Both stations calculate the altitude and divide by two.

Direct Measurement

Altitude = two trees,
28 concrete blocks,
or two windows



Here is a different methods for estimating altitude that is appropriate for lower grade students launching rockets that don't travel very high (e.g. straw rockets). Tracking students simply stand back and compare the rocket altitude to a building, tree, flagpole, etc.

A rough estimate of rocket altitude can also be made with a stopwatch. Time the total flight of the rocket and divide the time by 2. This yields the approximate time it took for the rocket to fall from its highest point back to the ground. The equation for falling bodies yields the altitude estimate. This method won't work if the rocket has a recovery system such as streamers or parachutes to slow its fall.

Sample Measurement:

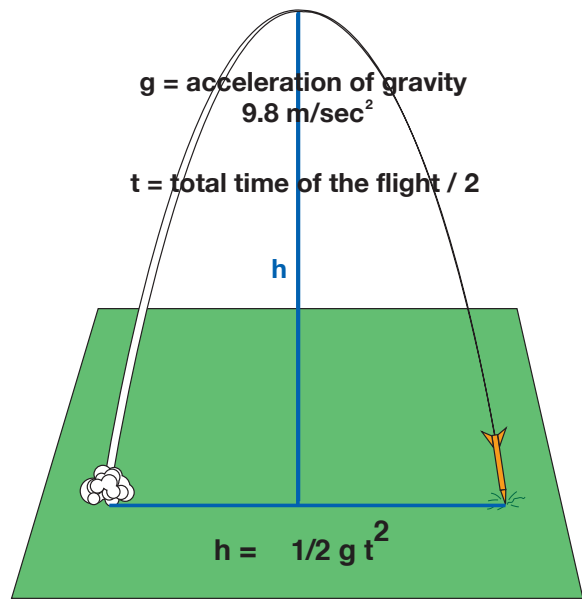
Total flight time: 6.2 seconds

Falling time/2 = 3.1 seconds

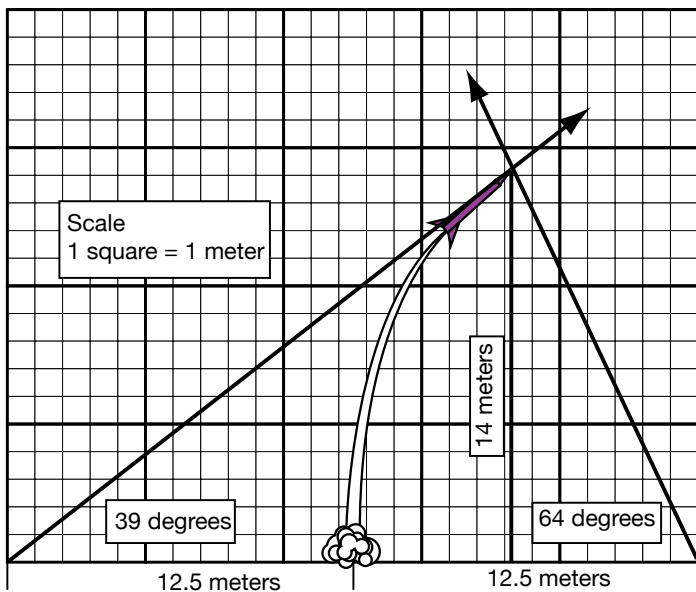
$$h = 1/2 g t^2$$

$$h = 1/2 \times 9.8 \text{ m} \times 9.6 \text{ (the seconds cancel out)}$$

$$h = 47.04 \text{ m}$$



Provides a rough estimate of the altitude reached. Air drag on the rocket is a significant source of error.



Here is a method for calculating altitude graphically. Two tracking stations are placed equidistant from the launcher. In this example, the stations are each 12.5 meters from the launcher. Both stations measure the angle. On a piece of graph paper, a scale drawing of the stations and the launch site is made. Using the principle of similar triangles, the scale altitude of the rocket is measured - 14 meters.

There is a considerably more advanced method for altitude tracking that also involves two tracking stations. The method not only requires measuring the altitude angle of the rocket but also its azimuth, or compass direction, from the tracking site. These two measurements from each station provide very accurate estimates of altitude regardless of how much the rocket drifts from the vertical. The problem with the method is that it requires a tracking device similar to a surveyor transit plus experienced trackers to take the measurements. Rocket hobbyists, especially those that participate in high performance rocketry, use small recording altimeters inside their rocket payload sections. These rockets are easily capable of flights of several thousand meters, and ground tracking stations have a hard time providing consistent and accurate data. Upon recovery, the altimeters are read. For more information on two-station tracking and altimeters, search the Internet for "rocket altitude tracking."