

Introduction

Space flight is important for many reasons. Space flight carries scientific instruments and human researchers high above the ground, permitting us to see Earth as a planet and to study the complex interactions of atmosphere, oceans, land, energy, and living things. Space flight lifts scientific instruments above the filtering effects of the atmosphere, making the entire **electromagnetic spectrum** available and allowing us to see more clearly the distant planets, stars, and galaxies. Space flight permits us to travel directly to other worlds to see them close up and sample their compositions. Finally, space flight allows scientists to investigate the fundamental states of matter—solids, liquids, and gases—and the **forces** that affect them in a microgravity environment.

The study of the states of matter and their interactions in microgravity is an exciting opportunity to expand the frontiers of science. Areas of investigation include biotechnology, combustion science, fluid physics, fundamental physics, materials science, and ways in which these areas of research can be used to advance efforts to explore the Moon and Mars.

Microgravity is the subject of this teacher's guide. This publication identifies the underlying mathematics, physics, and technology principles that apply to microgravity. Supplementary information is included in other NASA educational products.

First, What is Gravity?

Gravitational attraction is a fundamental property of matter that exists throughout the known universe. Physicists identify gravity as one of the four types of forces in the universe. The others are the strong and weak nuclear forces and the electromagnetic force.

Mathematics Standards

- Mathematical Connections
- Mathematics as Communication
- Δ Number and Number Relationships
- Δ Number Systems and Number Theory

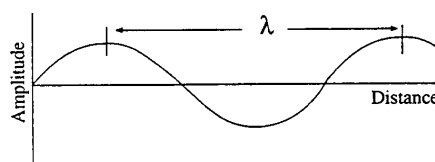
Science Standards

- Δ Physical Science
- Δ Unifying Concepts and Processes

The **electromagnetic spectrum** is generally separated into different radiation categories defined by frequency (units of Hertz) or wavelength (units of meters). Wavelength is commonly represented by the symbol λ .

Example:

Name	Approximate Wavelength (m)
Xrays	$= 10^{-15}$ to 10^{-9}
Ultraviolet	$= 10^{-8}$ to 10^{-7}
Visible Light	$= 10^{-7}$ to 10^{-6}
Infrared	$= 10^{-6}$ to 10^{-3}
Microwave	$= 10^{-3}$ to 10^{-1}
Television	$= 10^{-1}$ to 1
AM Radio	$= 10^{-2}$ to 10^3



Mathematics Standards

- Δ Algebra
- Conceptual Underpinnings of Calculus
- Geometry
- Geometry from an Algebraic Perspective
- Δ Mathematical Connections
- Δ Mathematics as Reasoning
- Trigonometry

Science Standards

- Δ Physical Science
- Δ Unifying Concepts and Processes

An impressed **force** is an action exerted upon a body, in order to change its state, either of rest, or of uni-



form motion in a straight line. A body force acts on the entire mass as a result of an external effect not due to direct contact; gravity is a body force. A surface force is a contact force that acts across an internal or external surface of a body.

Mathematics Standards

- Δ Algebra
- Conceptual Underpinnings of Calculus
- Δ Geometry
- Geometry from an Algebraic Perspective
- Δ Mathematical Connections
- Δ Mathematics as Reasoning
- Trigonometry

Science Standards

- Δ Physical Science
- Δ Unifying Concepts and Processes

Velocity is the rate at which the position of an object changes with time; it is a vector quantity. Speed is the magnitude of velocity.

Mathematics Standards

- Δ Mathematical Connections
- Δ Mathematics as Reasoning

Science Standards

- Δ History and Nature of Science
- Δ Science as Inquiry
- Δ Unifying Concepts and Processes

Newton's discovery of the universal nature of the **force of gravity** was remarkable. To take the familiar force that makes an apple fall to Earth and be able to recognize it as the same force that keeps the planets on their quiet and predictable paths represents one of the major achievements of human intellectual endeavor. This ability to see beyond the obvious and familiar is the mark of a true visionary. Sir Issac Newton's pioneering work epitomizes this quality.

Mathematics Standards

- Δ Algebra
- Δ Computation and Estimation
- Functions
- Δ Mathematical as Communication
- Δ Number and Number Relationships
- Δ Patterns and Functions

Science Standards

- Δ Unifying Concepts and Processes

More than 300 years ago the great English scientist Sir Isaac Newton published the important generalization that mathematically describes this universal force of gravity. Newton was the first to realize that gravity extends well beyond the domain of Earth. The basis of this realization stems from the first of three laws he formulated to describe the motion of objects. Part of Newton's first law, the law of inertia, states that objects in motion travel in a straight line at a constant **velocity** unless acted upon by a net force. According to this law, the planets in space should travel in straight lines. However, as early as the time of Aristotle, scholars knew that the planets travelled on curved paths. Newton reasoned that the closed orbits of the planets are the result of a net force acting upon each of them. That force, he concluded, **is the same force that causes an apple to fall to the ground—gravity.**

Newton's experimental research into the force of gravity resulted in his elegant mathematical statement that is known today as the Law of Universal Gravitation. According to Newton, every mass in the universe attracts every other mass. The attractive force between any two objects is **directly proportional** to the product of the two masses being considered and inversely proportional to the square of the distance separating them. If we let F represent this force, r represent the distance between the centers of the masses, and m_1 and m_2 represent the magnitudes of the masses, the relationship stated can be written symbolically as:

$$F \propto \frac{m_1 m_2}{r^2}$$

From this relationship, we can see that the greater the masses of the attracting objects, the greater the force of attraction between them. We can also see that the farther apart the objects are from each other, the less the attraction. If the distance between the objects doubles, the attraction between them diminishes by a factor of four, and if the distance triples, the attraction is only one-ninth as much.



The eighteenth-century English physicist Henry Cavendish later quantified Newton's Law of Universal Gravitation. He actually measured the gravitational force between two **one kilogram masses** separated by a distance of one meter. This attraction was an extremely weak force, but its determination permitted the proportional relationship of Newton's law to be converted into an equality. This measurement yielded the universal gravitational constant, G . Cavendish determined that the value of G is $6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2$. With G added to make the equation, the Law of Universal Gravitation becomes:

$$F = G \frac{m_1 m_2}{r^2}$$

What is Microgravity?

The presence of Earth creates a gravitational field that acts to attract objects with a force inversely proportional to the square of the distance between the center of the object and the center of Earth. When we measure the acceleration of an object acted upon only by Earth's gravity at the Earth's surface, we commonly refer to it as one g or one Earth gravity. This acceleration is approximately 9.8 meters per second squared (m/s^2). The mass of an object describes how much the object accelerates under a given force. The weight of an object is the gravitational force exerted on it by Earth. In British units (commonly used in the United States), force is given in units of pounds. The British unit of mass corresponding to one pound force is the slug.

While the mass of an object is constant and the weight of an object is constant (ignoring differences in g at different locations on the Earth's surface), the environment of an object may be changed in such a way that its apparent weight changes. Imagine standing on a scale in a stationary elevator car. Any vertical accelerations of the elevator are considered to be positive

$F \propto \frac{m_1 m_2}{r^2}$ indicates **proportionality**

$F = G \frac{m_1 m_2}{r^2}$ indicates equality

$G \frac{m_1 m_2}{r^2}$ is an expression

$F = G \frac{m_1 m_2}{r^2}$ is an equation

Mathematics Standards

- Δ Algebra
- Δ Mathematical Connections
- Δ Mathematics as Communication
- Δ Measurement

Science Standards

- Δ Science and Technology
- Δ Science as Inquiry
- Δ Unifying Concepts and Processes

The internationally recognized Systeme International (SI) is a system of measurement units. The SI units for length (meter) and **mass (kg)** are taken from the metric system. Many dictionaries and mathematics and science textbooks provide conversion tables between the metric system and other systems of measurement. Units conversion is very important in all areas of life, for example in currency exchange, airplane navigation, and scientific research.

Units Conversion Examples

1 kg \cong 2.2lb 1 in = 2.54cm
1 liter \cong 1 qt 1 yd \cong 0.9 m

Questions for Discussion

- What common objects have a mass of about 1 kg?
- What are the dimensions of this sheet of paper in cm and inches?
- How many liters are there in a gallon?

Mathematics Standards

- Δ Computation and Estimation
- Δ Mathematics as Communication
- Δ Number and Number Relationships
- Δ Number Systems and Number Theory



Science Standards

- Δ Science as Inquiry
- Δ Science in Personal and Social Perspectives
- Δ Unifying Concepts and Processes

Scientific notation makes it easier to read, write, and manipulate numbers with many digits. This is especially useful for making quick estimates and for indicating the number of significant figures.

Examples:

$$\begin{aligned}0.001 &= 10^{-3} \\ 10 &= 10^1 \\ 1000 &= 10^3\end{aligned}$$

Which is bigger, 6×10^{-3} or 8×10^{-4} ? 6×10^3 or 8×10^4 ?
How much bigger?

Mathematics Standards

- Δ Mathematical Connections
- Δ Mathematics as Reasoning

Science Standards

- Δ Science and Technology
- Δ Science as Inquiry
- Δ Unifying Concepts and Processes

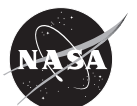
Questions for Discussion

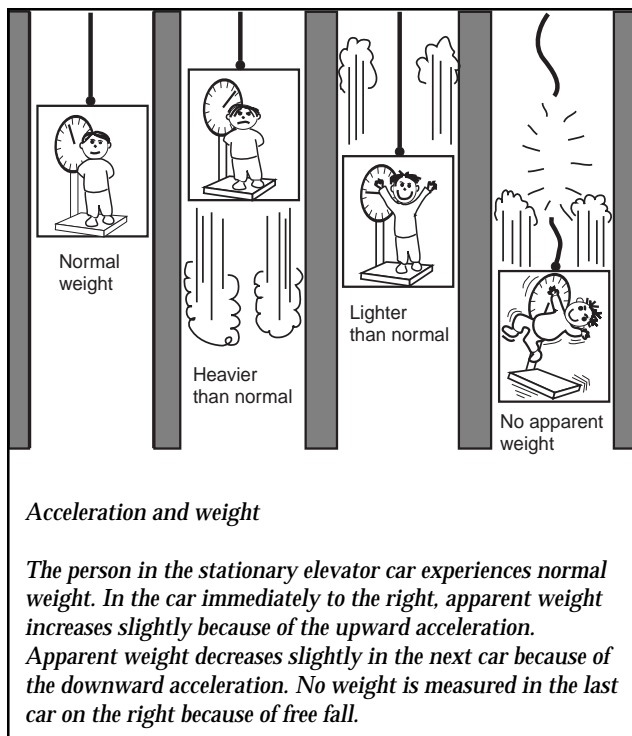
- How does a **scale** work ?
- What does a scale measure?
- How many different kinds of scales can you list?
- Do they need gravity for them to work?
- Would you get different results on the Moon or Mars?
- How can you measure the mass of an object in microgravity?

upwards. Your weight, **W**, is determined by your mass and the acceleration due to gravity at your location.

If you begin a ride to the top floor of a building, an additional force comes into play due to the acceleration of the elevator. The force that the floor exerts on you is your apparent weight, **P**, the magnitude of which the **scale** will register. The total force acting on you is $\mathbf{F}=\mathbf{W}+\mathbf{P}=\mathbf{ma}_e$, where \mathbf{a}_e is the acceleration of you and the elevator and $\mathbf{W}=\mathbf{mg}$. Two example calculations of apparent weight are given in the margin of the next page. Note that if the elevator is not accelerating then the magnitudes **W** and **P** are equal but the direction in which those forces act are opposite ($\mathbf{W}=-\mathbf{P}$). Remember that the sign (positive or negative) associated with a vector quantity, such as force, is an indication of the direction in which the vector acts or points, with respect to a defined frame of reference. For the reference frame defined above, your weight in the example in the margin is negative because it is the result of an acceleration (gravity) directed downwards (towards Earth).

Imagine now riding in the elevator to the top floor of a very tall building. At the top, the cables supporting the car break, causing the car and you to fall towards the ground. In this example, we discount the effects of air friction and elevator safety mechanisms on the falling car. Your apparent weight $\mathbf{P}=\mathbf{m}(\mathbf{a}_e-\mathbf{g})=(60\text{ kg})(-9.8\text{ m/s}^2-(-9.8\text{ m/s}^2))=0\text{ kg m/s}^2$; you are weightless. The elevator car, the scale, and you would all be accelerating downward at the same rate, which is due to gravity alone. If you lifted your feet off the elevator floor, you would float inside the car. This is the same experiment that Galileo is purported to have performed at Pisa, Italy, when he dropped a cannonball and a musketball of different mass at the same time from the same height. Both balls hit the ground at the same time, just as the elevator car, the scale, and you would reach the ground at the same time.





For reasons that are discussed later, there are many advantages to performing scientific experiments under conditions where the apparent weight of the experiment system is reduced. The name given to such a research environment is microgravity. The prefix micro- (m) derives from the original Greek mikros meaning small. By this definition, **a microgravity environment is one in which the apparent weight of a system is small compared to its actual weight due to gravity.** As we describe how microgravity environments can be produced, bear in mind that many factors contribute to the experienced accelerations and that the quality of the microgravity environment depends on the mechanism used to create it. In practice, the microgravity environments used by scientific researchers range from about one percent of Earth's gravitational acceleration (aboard aircraft in parabolic flight) to better than one part in a million (for example, onboard Earth-orbiting research satellites).

Quantitative systems of measurement, such as the metric system, commonly use **micro-** to mean one part in a million. Using that definition, the acceleration experienced by an object in a

Mathematics Standards

- Δ Algebra
- Δ Computational and Estimation
- Conceptual Underpinnings of Calculus
- Δ Mathematical Connections
- Δ Mathematics as Problem Solving
- Δ Measurement

Science Standards

- Δ Physical Science
- Δ Science and Technology
- Δ Science as Inquiry
- Δ Unifying Concepts and Processes

$$F=W+P=ma_c$$

Rewriting yields $P=ma_c - mg=m(a_c - g)$.

If your mass is 60 kg and the elevator is accelerating upwards at 1 m/s², your apparent weight is

$$P=60 \text{ kg} (+1 \text{ m/s}^2 - (-9.8 \text{ m/s}^2)) = +648 \text{ kg m/s}^2$$

while your weight remains

$$W=mg=(60 \text{ kg})(-9.8 \text{ m/s}^2)=-588 \text{ kg m/s}^2.$$

If the elevator accelerates downwards at 0.5 m/s², your apparent weight is

$$P=60 \text{ kg} (-0.5 \text{ m/s}^2 - (-9.8 \text{ m/s}^2)) = +558 \text{ kg m/s}^2.$$

Mathematics Standards

- Δ Mathematics as Communications
- Δ Mathematics as Reasoning

Science Standards

- Δ Science as Inquiry
- Δ Science in Personal and Social Perspectives
- Δ Unifying Concepts and Processes

$$1 \text{ micro-g or } 1 \mu\text{g} = 1 \times 10^{-6} \text{ g}$$

Questions for Discussion

- What other common prefixes or abbreviations for powers of ten do you know or can you find?
- In what everyday places do you see these used?
Grocery stores, farms, laboratories, sporting facilities, pharmacies, machine shops.

Common prefixes for powers of ten:

10 ⁻⁹	nano-	n
10 ⁻³	milli-	m
10 ²	centi-	c
10 ³	kilo-	k
10 ⁶	mega-	M
10 ⁹	giga-	G



Mathematics Standards

- Δ Algebra
- Δ Computation and Estimation
 - Conceptual Underpinnings of Calculus
 - Discrete Mathematics
- Δ Mathematical Connections
- Δ Mathematics as Problem Solving
- Δ Mathematics as Reasoning
- Δ Number and Number Relationships

Science Standards

- Δ Unifying Concepts and Processes

Calculate the times in these **examples**. Teachers can use these examples at several different scholastic levels.

Provide the equation as:

$$t = \sqrt{\frac{2d}{a}} \text{ or } \left(\frac{2d}{a}\right)^{1/2}$$

Provide the equation as $d=(1/2)at^2$, and have the students re-order the equation.

Making measurements and calculating results involve the concepts of accuracy and precision, significant figures, and orders of magnitude. With these concepts in mind, are the drop times given in the text “correct”?

Mathematics Standards

- Δ Algebra
- Δ Computation and Estimation
- Δ Mathematical Connections
- Δ Mathematics as Problem Solving
- Δ Mathematics as Reasoning
- Δ Measurement

Science Standards

- Δ Science and Technology
- Δ Science as Inquiry
- Δ Unifying Concepts and Processes

Questions for Discussion

- **How far away is the Moon?**
- How far away is the center of Earth from the center of the Moon?
- Why did we ask the previous question?
- How far away is the surface of Earth from the surface of the Moon?
- What are the elevations of different features of Earth and the Moon?
- How are elevations measured?

microgravity environment would be one-millionth (10^{-6}) of that experienced at Earth’s surface. The use of the term microgravity in this guide will correspond to the first definition. For illustrative purposes only, we provide the following simple example using the quantitative definition. This **example** attempts to provide insight into what might be expected if the local acceleration environment would be reduced by six orders of magnitude from 1 g to 10^{-6} g,

If you dropped a rock from a roof that was **five meters** high, it would take just **one second** to reach the ground. In a reduced gravity environment with **one percent** of Earth’s gravitational pull, the same drop would take **10 seconds**. In a microgravity environment equal to **one-millionth** of Earth’s gravitational pull, the same drop would take **1,000 seconds** or about **17 minutes!**

Researchers can create microgravity conditions in two ways. Because gravitational pull diminishes with distance, one way to create a microgravity environment (following the quantitative definition) is to travel away from Earth. To reach a point where Earth’s gravitational pull is reduced to onemillionth of that at the surface, you would have to travel into space a distance of **6.37 million kilometers from Earth (almost 17 times farther away than the Moon, 1400 times the highway distance between New York City and Los Angeles, or about 70 million football fields)**. This approach is impractical, except for automated spacecraft, because humans have yet to travel farther away from Earth than the distance to the Moon. However, freefall can be used to create a microgravity environment consistent with our primary definition of microgravity. We discuss this in the next section.



Creating Microgravity

As illustrated in the elevator examples in the previous section, the effects of gravity (apparent weight) can be removed quite easily by putting anything (a person, an object, an experiment) into a state of freefall. This possibility of using Earth's gravity to remove the effects of gravity within a system were not always evident. Albert Einstein once said, "I was sitting in a chair in the patent office at Bern when all of a sudden a thought occurred to me: 'If a person falls freely, he will not feel his own weight.' I was startled. This simple thought made a deep impression on me. It impelled me toward a theory of gravitation." Working with this knowledge, scientists involved in early space flights rapidly concluded that micro-gravity experiments could be performed by crew members while in orbit.

Gravity and Distance

The inverse square relationship between gravitational force and distance can be used to determine the acceleration due to gravity at any distance from the center of Earth, r .

$$F = Gm_e m / r_e^2 \quad \text{force of gravity due to Earth on a mass, } m, \text{ at Earth's surface}$$

$$F = mg \rightarrow g = Gm_e / r_e^2$$
$$F = Gm_e m / r^2 \quad \text{force of gravity due to Earth on a mass, } m, \text{ at a distance, } r, \text{ from Earth's center}$$

$$F = ma \rightarrow a = Gm_e / r^2$$
$$gr_e^2 = ar^2$$
$$a = gr_e^2 / r^2 \quad \text{acceleration due to Earth's gravity at distance, } r, \text{ from Earth's center}$$

A typical altitude for a Space Shuttle Orbiter orbit is 296 km. The Earth's mean radius is 6.37×10^6 m. The acceleration due to gravity at the Orbiter's altitude is

$$a = 9.8 \text{ m/s}^2 (6.37 \times 10^6 \text{ m})^2 / (6.67 \times 10^6 \text{ m})^2 = 8.9 \text{ m/s}^2$$

This is about 90% of the acceleration due to gravity at Earth's surface. Using the same equations, you can see that to achieve a microgravity environment of $10^{-6} g$ by moving away from Earth, a research laboratory would have to be located 6.37×10^9 m from the center of Earth.



Mathematics Standards

- Δ Algebra
- Δ Computation and Estimation
 - Conceptual Underpinnings of Calculus
 - Discrete Mathematics
 - Functions
- Δ Mathematical Connections
- Δ Mathematics as Problem Solving
- Δ Mathematics as Reasoning
- Δ Patterns and Functions
- Δ Statistics

Science Standards

- Δ Physical Science
- Δ Science and Technology
- Δ Science as Inquiry
- Δ Science in Personal and Social Perspectives
- Δ Unifying Concepts and Processes

Questions for Discussion

- What is the functional relationship between acceleration, distance, and time?

Use the four sets of **drop facility data points** given in the text and the additional data set (0 meters, 0 seconds). What does the (0 meters, 0 seconds) data set represent? Why is it a valid data set to use?

Suggested solution methods: Use different types of graph paper Use a computer e urvefitting r)rogram Do a dimensional analysis.

- Knowing that $g=9.8 \text{ m/s}^2$, what equation can you write to incorporate acceleration, distance, and time?
- Assume it costs \$5,000 per meter of height to build a drop tower.

How much does it cost to build a drop tower to allow drops of 1 second, 2 seconds, 4 seconds, 10 seconds?

Why does it cost so much more for the longer times?

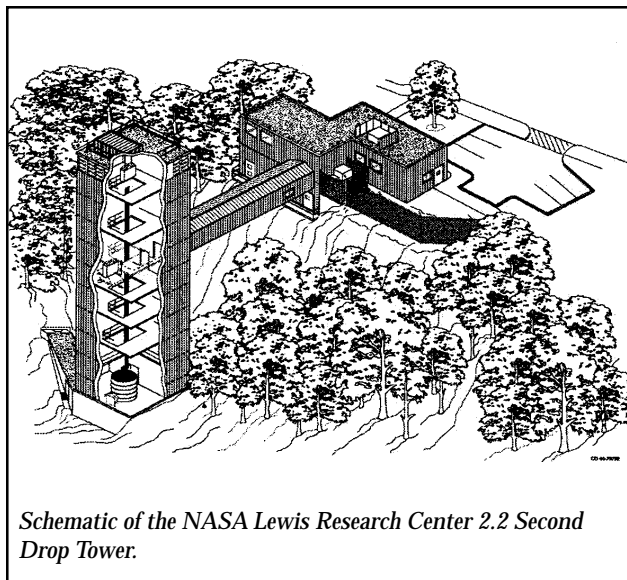
What would be an inexpensive way to double low-gravity time in a drop tower?

Shoot the experiment package up from the bottom.

The use of orbiting spacecraft is one method used by NASA to create microgravity conditions. In addition, four other methods of creating such conditions are introduced here and we give examples of situations where the student can experience microgravity.

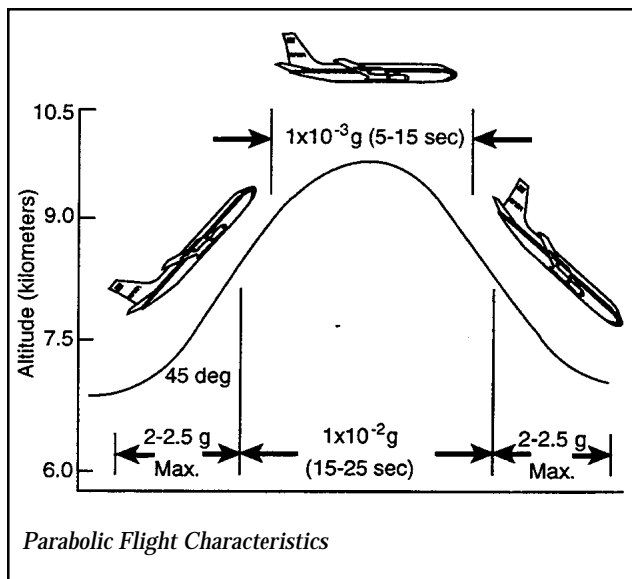
Drop Facilities

Researchers use high-tech facilities based on the elevator analogy to create micro-gravity conditions. The NASA Lewis Research Center has two drop facilities. One provides a **132 meter** drop into a hole in the ground similar to a mine shaft. This drop creates a reduced gravity environment for **5.2 seconds**. A tower at Lewis allows for 2.2 second drops down a **24 meter** structure. The NASA Marshall Space Flight Center has a different type of reduced gravity facility. This **100 meter** tube allows for drops of **4.5 second** duration. Other NASA Field Centers and other countries have additional drop facilities of varying sizes to serve different purposes. The longest drop time currently available (about **10 seconds**) is at a **490 meter** deep vertical mine shaft in Japan that has been converted to a drop facility. Sensations similar to those resulting from a drop in these reduced gravity facilities can be experienced on freefall rides in amusement parks or when stepping off of diving platforms.



Aircraft

Airplanes are used to achieve reduced gravity conditions for periods of about 15 seconds. This environment is created as the plane flies on a parabolic path. A typical flight lasts two to three hours allowing experiments and crew members to take advantage of about forty periods of microgravity. To accomplish this, the plane climbs rapidly at a 45 degree angle (this phase is called pull up), **traces a parabola** (pushover), and then descends at a 45 degree angle (pull out). During the pull up and pull out segments, crew and experiments experience accelerations of about 2 g. During the parabola, net accelerations drop as low as $1.5 \times 10^{-2} g$ for about 15 seconds. Due to the experiences of many who have flown on parabolic aircraft, the planes are often referred to as "Vomit Comets." Reduced gravity conditions created by the same type of parabolic motion described above can be experienced on the series of "floater" hills that are usually located at the end of roller coaster rides and when driving over swells in the road.



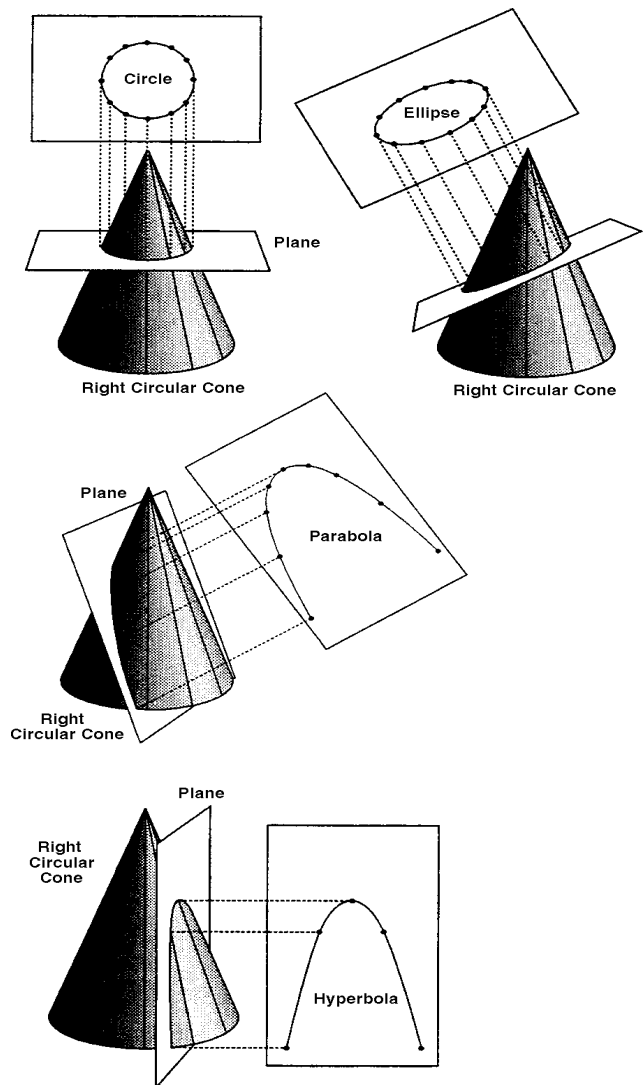
Mathematics Standards

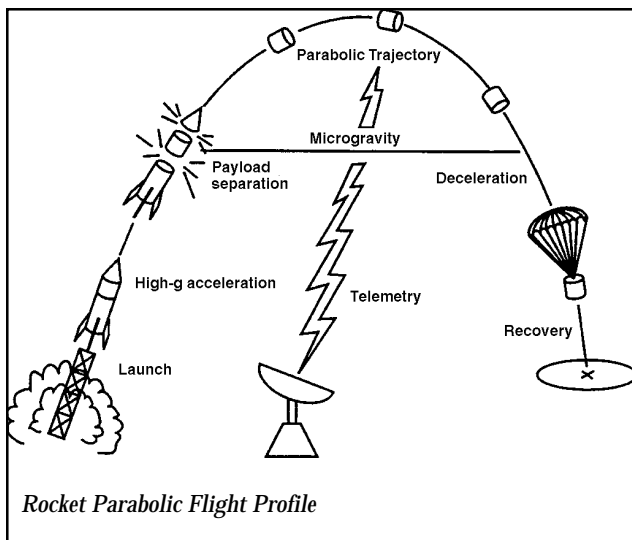
- Conceptual Underpinnings of Calculus
- Functions
- Δ Mathematical Connections
- Δ Patterns and Functions

Science Standards

- Δ Earth and Space Science
- Δ Physical Science
- Δ Unifying Concepts and Processes

Microgravity carriers and other spacecraft follow paths best described by conic sections. The aircraft and sub-orbital rockets trace out **parabolas**. Orbiting spacecraft are free falling on elliptical paths. When a meteoroid is on a path that is influenced by Earth or any other planetary body but does not get captured by the gravitational field of the body, its motion, as it approaches then moves away from the body, traces out a hyperbolic path.





Rockets

Sounding rockets are used to create reduced gravity conditions for several minutes; they follow suborbital, parabolic paths. Freefall exists during the rocket's coast: after burn out and before entering the atmosphere. Acceleration levels are usually around 10^{-5} g. While most people do not get the opportunity to experience the accelerations of a rocket launch and subsequent freefall, springboard divers basically launch themselves into the air when performing dives and they experience microgravity conditions until they enter the water.

Orbiting Spacecraft

Although drop facilities, airplanes, and rockets can establish a reduced gravity environment, all these facilities share a common problem. After a few seconds or minutes, Earth gets in the way and freefall stops. To conduct longer scientific investigations, another type of freefall is needed.

To see how it is possible to establish microgravity conditions for long periods of time, one must first understand what keeps a spacecraft in orbit. Ask any group of students or adults **what keeps satellites and Space Shuttles in orbit** and you will probably get a variety of answers. Two common answers are "The rocket engines keep firing to hold it up," and "There is no gravity in space."

Although the first answer is theoretically possible, the path followed by the spacecraft would technically not be an orbit. Other than the altitude involved and the specific means of exerting an upward force, little difference exists between a spacecraft with its engines constantly firing and an airplane flying around the world. A satellite could not carry enough fuel to maintain its altitude for more than a few minutes. The second answer is also wrong. At the altitude that the Space Shuttle typically orbits Earth, the gravitational pull on the Shuttle by Earth is about 90% of what it is at Earth's surface.

Mathematics Standards

- Δ Algebra
- Δ Computation and Estimation
 - Conceptual Underpinnings of Calculus
 - Discrete Mathematics
 - Functions
- Δ Mathematical Connections
- Δ Mathematics as Problem Solving
- Δ Mathematics as Reasoning
- Δ Number and Number Relationships

Science Standards

- Physical Science
- Δ Science and Technology
 - Science as Inquiry
 - Unifying Concepts and Processes

Questions for Discussion

- **How does the Shuttle stay in orbit?** Use the following two equations that describe the force acting on an object. The first equation represents the force of gravity acting on the Shuttle.

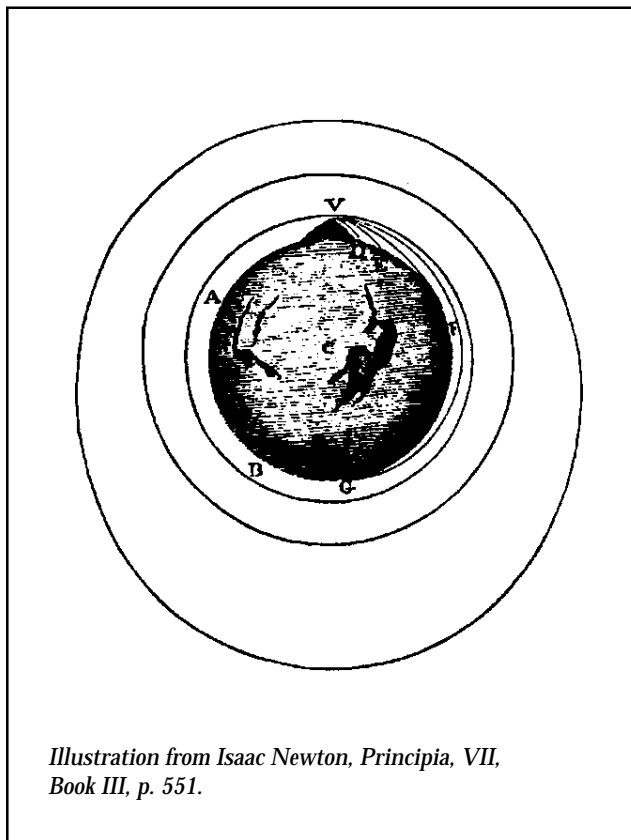
$$F_1 = G \frac{m_e m_s}{r^2}$$

Where:

- F_1 = Force of gravity acting on the Shuttle
- G = Universal gravitational constant
- m_e = Mass of Earth
- m_s = Mass of the Shuttle
- r = Distance from center of Earth to the Shuttle



In a previous section, we indicated that Issac Newton reasoned that the closed orbits of the planets through space were due to gravity's presence. Newton expanded on his conclusions about gravity and hypothesized how an artificial satellite could be made to orbit Earth. He envisioned a very tall mountain extending above Earth's atmosphere so that friction with the air would not be a factor. He then imagined a cannon at the top of that mountain firing cannonballs parallel to the ground. Two forces acted upon each cannonball as it was fired. One force, due to the explosion of the black powder, propelled the cannonball straight outward. If no other force were to act on the cannonball, the shot would travel in a straight line and at a constant velocity. But Newton knew that a second force would act on the cannonball: gravity would cause the path of the cannonball to bend into an arc ending at Earth's surface.



The second equation represents the force acting on the Shuttle that causes a centripetal acceleration,

$$\frac{v^2}{r}$$

This is an expression of Newton's second law, $F=ma$.

- F_2 = Force acting on the Shuttle that causes uniform circular motion (with centripetal acceleration)
- v = Velocity of the Shuttle

These two forces are equal: $F_1=F_2$

$$G \frac{m_e m_s}{r^2} = \frac{m_s v^2}{r}$$

$$v^2 = \frac{Gm_e}{r}$$

$$v = \sqrt{\frac{Gm_e}{r}}$$

In order to stay in a circular orbit at a given distance from the center of Earth, r , the Shuttle must travel at a precise velocity, v .

- How does the Shuttle change its **altitude**? From a detailed equation relating the Shuttle velocity with the Shuttle altitude, one can obtain the following simple relationship for a circular orbit. Certain simplifying assumptions are made in developing this equation: 1) the radius of the Shuttle orbit is nearly the same as the radius of Earth, and 2) the total energy of the Shuttle in orbit is due to its kinetic energy, $1/2 mv^2$; the change in potential energy associated with the launch is neglected.

$$\Delta r = \frac{\tau}{\pi} \Delta v$$

τ = orbital period. the time it takes the Shuttle to complete one revolution around Earth

$$= \frac{2 \pi r^{3/2}}{(Gm_e)^{1/2}}$$

Δv = the change in Shuttle velocity

Δr = the change in Shuttle altitude



For example:

Consider a Shuttle in a circular orbit at 160 nautical miles (296.3 km) altitude. Determine the new altitude caused by the Shuttle firing a thruster that increases its velocity by 1 m/s.

First, calculate the orbital period, X , from the above equation.

$$\begin{aligned}\tau &= \frac{2\pi(re+2.96 \times 10^5 \text{ m})^{3/2}}{(Gm_e)^{1/2}} \\ &= \frac{2\pi(6.37 \times 10^6 \text{ m} + 2.96 \times 10^5 \text{ m})^{3/2}}{(6.67 \times 10^{-11} \frac{\text{m}^3}{\text{s}^2 \text{kg}} \times 5.98 \times 10^{24} \text{ kg})^{1/2}} \\ &= 5.41 \times 10^3 \text{ s}\end{aligned}$$

Next, use the period and the applied velocity change to calculate the altitude change.

$$\begin{aligned}\Delta r &= \frac{\tau}{\pi} \Delta v \\ &= \frac{5.41 \times 10^3 \text{ s}}{\pi} (1 \text{ m/s}) \\ &= 1.72 \times 10^3 \text{ m}\end{aligned}$$

This altitude change is actually seen on the opposite side of the orbit. In order to make the orbit circular at the new altitude, the Shuttle needs to apply the same Δv at the other side of the orbit.

In the discussion and example just given, we state that the equations given are simple approximations of more complex relationships between Shuttle velocity and altitude. The more complex equations are used by the Shuttle guidance and navigation teams who track the Shuttles' flights. But the equations given here can be used for quick approximations of the types of thruster firings needed to achieve certain altitude changes. This is helpful when an experiment team may want to request an altitude change. Engineers supporting the experiment teams can determine approximately how much propellant would be required for such an altitude change and whether enough would be left for the required de-orbit burns. In this way, the engineers and experiment teams can see if their request is realistic and if it has any possibility of being implemented.

Newton considered how additional cannonballs would travel farther from the mountain each time the cannon fired using more black powder. With each shot, the path would lengthen and soon the cannonballs would disappear over the horizon. Eventually, if one fired a cannon with enough energy, the cannonball would fall entirely around Earth and come back to its starting point. The cannonball would be in orbit around Earth. Provided no force other than gravity interfered with the cannonball's motion, it would continue circling Earth in that orbit.

This is how the Space Shuttle stays in orbit. It launches on a path that arcs above Earth so that the Orbiter travels at the right speed to keep it falling while maintaining a constant altitude above the surface. For example, if the Shuttle climbs to a 320 kilometer high orbit, it must travel at a speed of about 27,740 kilometers per hour to achieve a stable orbit. **At that speed and altitude**, the Shuttle executes a falling path parallel to the curvature of Earth. Because the Space Shuttle is in a state of freefall around Earth and due to the extremely low friction of the upper atmosphere, the Shuttle and its contents are in a high-quality microgravity environment.

